The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Topic Generator - Problem Set
Problems

1. The value of $8 + 2(3^2)$ is
   (A) 26 (B) 90 (C) 41 (D) 44 (E) 60

2. What is the value of $(-1)^5 - (-1)^4$?
   (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

3. Which is the largest sum?
   (A) $\frac{1}{4} + \frac{1}{5}$ (B) $\frac{1}{4} + \frac{1}{6}$ (C) $\frac{1}{4} + \frac{1}{3}$ (D) $\frac{1}{4} + \frac{1}{8}$ (E) $\frac{1}{4} + \frac{1}{7}$

4. The value of $10^2 + 10 + 1$ is
   (A) 101 (B) 1035 (C) 1011 (D) 111 (E) 31

5. If there is no tax, which of the following costs more than \$18 to purchase?
   (A) Five \$1 items and five \$2 items
   (B) Nine \$1 items and four \$2 items
   (C) Nine \$1 items and five \$2 items
   (D) Two \$1 items and six \$2 items
   (E) Sixteen \$1 items and no \$2 items

6. An electric car is charged 3 times per week for 52 weeks. The cost to charge the car each time is \$0.78. What is the total cost to charge the car over these 52 weeks?
   (A) \$104.00 (B) \$81.12 (C) \$202.80 (D) \$162.24 (E) \$121.68

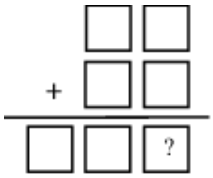
7. The expression $2 \times 0 + 1 - 9$ equals
   (A) -8 (B) -6 (C) -7 (D) -11 (E) 0

8. In a sequence of numbers, the first term is 3. Each new term is obtained by adding 5 to the previous term. The first four terms are $3, 8, 13, 18$. What are the next three terms in the sequence?
(A) $25, 30, 35$ (B) $5, 10, 15$ (C) $23, 28, 33$ (D) $23, 33, 43$ (E) $19, 20, 21$

9. A mother bear collects 14 fish. She gives 4 fish to each of her 3 bear cubs. How many fish does the mother bear have left over?
(A) 0 (B) 2 (C) 3 (D) 4 (E) 5

10. If $50 - 2\sqrt{x} = 18$, the value of x is
(A) 32 (B) 16 (C) 64 (D) 256 (E) 8

11. In the addition of two 2-digit numbers, each blank space, including those in the answer, is to be filled with one of the digits 0, 1, 2, 3, 4, 5, 6, each used exactly once. The units digit of the sum is



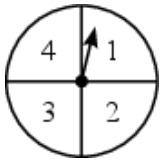
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

12. If x and y are two-digit positive integers with $xy = 555$, what is $x + y$?
(A) 52 (B) 116 (C) 66 (D) 555 (E) 45

13. The sum of four numbers is T. Suppose that each of the four numbers is now increased by 1. These four new numbers are added together and then the sum is tripled. What is the value of this final result?
(A) $3T + 3$ (B) $3T + 4$ (C) $3T + 12$ (D) $T + 12$ (E) $12T$

14. The operation ∇ is defined by $g \nabla h = g^2 - h^2$. For example, $2 \nabla 1 = 2^2 - 1^2 = 3$. If $g > 0$ and $g \nabla 6 = 45$, the value of g is
(A) 39 (B) 6 (C) 81 (D) 3 (E) 9

15. On each spin of the spinner shown, the arrow is equally likely to stop on any one of the four numbers. Deanna spins the arrow on the spinner twice. She multiplies together the two numbers on which the arrow stops. Which product is most likely to occur?



(A) 2 (B) 4 (C) 6 (D) 8 (E) 12

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16. The operation $\otimes$ is defined by $a \otimes b = \frac{a}{b} + \frac{b}{a}$. What is the value of $4 \otimes 8$?

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{5}{4}$ (D) 2 (E) $\frac{5}{2}$

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17. In the sum shown, P and Q each represent a digit. The value of $P + Q$ is

$$\begin{array}{r} PQQ \\ PPQ \\ +\,QQQ \\ \hline 876 \end{array}$$

(A) 3 (B) 5 (C) 7 (D) 6 (E) 4

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18. The mean (average) height of a group of children would be increased by 6 cm if 12 of the children in the group were each 8 cm taller. How many children are in the group?

(A) 16 (B) 14 (C) 21 (D) 26 (E) 9

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19. Janet picked a number, added 7 to the number, multiplied the sum by 2, and then subtracted 4. If the final result was 28, what number did Janet pick?

(A) 9 (B) 5 (C) 19 (D) 23 (E) 11

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20. Mateo's 300 km trip from Edmonton to Calgary passed through Red Deer. Mateo started in Edmonton at 7 a.m. and drove until stopping for a 40 minute break in Red Deer. Mateo arrived in Calgary at 11 a.m. Not including the break, what was his average speed for the trip?
(A) 83 km/h (B) 94 km/h (C) 90 km/h (D) 95 km/h (E) 64 km/h

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21. A positive integer is called a *perfect power* if it can be written in the form a^b, where a and b are positive integers with $b \geq 2$. For example, 32 and 125 are perfect powers because $32 = 2^5$ and $125 = 5^3$. The increasing sequence

$$2, 3, 5, 6, 7, 10, \ldots$$

consists of all positive integers which are not perfect powers. The sum of the squares of the digits of the 1000th number in this sequence is
(A) 42 (B) 26 (C) 33 (D) 18 (E) 21

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22. The average of four different positive whole numbers is 4. If the difference between the largest and smallest of these numbers is as large as possible, what is the average of the other two numbers?
(A) $1\frac{1}{2}$ (B) $2\frac{1}{2}$ (C) 4 (D) 5 (E) 2

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23. A *Fano table* is a table with three columns where
    - each entry is an integer taken from the list $1, 2, 3, \ldots, n$, and
    - each row contains three different integers, and
    - for each possible pair of distinct integers from the list $1, 2, 3, \ldots, n$, there is exactly one row that contains both of these integers.

    The number of rows in the table will depend on the value of n. For example, the table shown is a Fano table with $n = 7$. (Notice that 2 and 6 appear in the same row only once, as does every other possible pair of the numbers $1, 2, 3, 4, 5, 6, 7$.) For how many values of n with $3 \leq n \leq 12$ can a Fano table be created?

1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

(A) 2 (B) 3 (C) 5 (D) 6 (E) 7

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24. A bicycle at Store P costs \$200. The regular price of the same bicycle at Store Q is 15% more than it is at Store P. The bicycle is on sale at Store Q for 10% off of the regular price. What is the sale price of the bicycle at Store Q?
(A) \$230.00 (B) \$201.50 (C) \$199.00 (D) \$207.00 (E) \$210.00

25. In her last basketball game, Jackie scored 36 points. These points raised the average (mean) number of points that she scored per game from 20 to 21. To raise this average to 22 points, how many points must Jackie score in her next game?
(A) 38 (B) 22 (C) 23 (D) 36 (E) 37

26. There are n students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the n students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the n students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the n students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to $n^2 - n$ is
(A) 11 (B) 12 (C) 20 (D) 13 (E) 10

27. A positive integer n with $n \geq 3$ is called a *Nella number* if there exists a positive integer x with $x < n$ and there exists a positive integer m such that
    - m is not divisible by x or by $x + 1$, and
    - m is divisible by every other positive integer between 1 and n inclusive.

    For example, $n = 7$ is a Nella number. How many Nella numbers n are there with $50 \leq n \leq 2017$?
(A) 393 (B) 394 (C) 395 (D) 396 (E) 397

28. In the multiplication shown, each of P, Q, R, S, and T is a digit. The value of $P + Q + R + S + T$ is
(A) 14 (B) 20 (C) 16
(D) 17 (E) 13

$$\begin{array}{r} P\,Q\,R\,S\,T\,4 \\ \times \qquad\quad 4 \\ \hline 4\,P\,Q\,R\,S\,T \end{array}$$

29. Consider positive integers $a \leq b \leq c \leq d \leq e$. There are N lists a, b, c, d, e with a mean of 2023 and a median of 2023, in which the integer 2023 appears more than once, and in which no other integer appears more than once. What is the sum of the digits of N?

30. Three circles have radii 1 cm, 5 cm, and x cm. If the mean (average) area of the three circles is $30\pi cm^2$, the value of x is
(A) 64 (B) 5 (C) 24 (D) 8 (E) 2